# Survey of Electroweak Interactions (lecture notes)

## Electromagnetism

Electromagnetism is a glorious subject. In its quantum form, it provides the firm foundation of atomic physics, chemistry, materials science, large parts of astrophysics and, of course, electronics and electrical engineering. Having said that, I will not give a self-contained survey of electromagnetism here, since it is covered in other courses. I'll only very briefly survey (or just mention) the aspects of the subject with direct connections to the formulation of the Standard Model and modern particle physics.

That still leaves a lot, since many of the foundational ideas in particle physics have their roots in electromagnetism. The list includes:

1. special relativity
2. gauge invariance
3. the association of fields with particles (photon) → quantization of fields
4. particles can be created and destroyed (emission and absorption of photons/light)
5. creation and destruction of particles is associated with nonlinear interactions of fields
6. the Dirac equation
7. quantization of (non-classical) fermion fields

Those ideas were all in place by the early 1930s. They supported an excellent treatment of atomic spectroscopy and of radiation processes, including novel processes like electron-positron pair creation. In these applications, it was generally sufficient to calculate to the lowest non-trivial order in the fine structure constant $\alpha$, since the experiments were no more accurate than that. Attempts to calculate corrections were plagued by mathematical ambiguities and divergent integrals, in any case.

Wartime developments in microwave technology and electronics enabled a new level of accuracy in atomic spectroscopy, and revealed small but finite discrepancies with the lowest-order calculations. These included a difference between the measured $g$-factor for the electron's magnetic dipole moment and the value $g = 2$ suggested by the minimal Dirac equation, and a splitting between the $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ levels of hydrogen, which the minimal Dirac theory predicts to be degenerate (Lamb shift). The magnitude of these discrepancies was consistent, at a semi-quantitative level, with what one might expect from order $\alpha$ corrections. These developments encouraged theorists to adopt a "radically conservative" approach to quantum electrodynamics – that is, to explore the existing theory fully, and to try to work through its apparent difficulties, before turning to modifications.

The result of this re-consideration, primarily through the work of Schwinger, Feynman, Tomonaga, and Dyson, was the formulation of renormalized perturbation theory for quantum electrodynamics (QED). The strategy of renormalization theory is as follows. To *regulate* the otherwise divergent integrals, one introduces some sort of cutoff. It is desirable for the cutoff to respect as much of the symmetry one ultimately hopes to embody as possible. Useful cutoffs include: dimensional regularization, which defines the integrals in a smaller number $d$ of space-time dimensions from $3 + 1$, where they converge, and then analytically continues the resulting expressions up; Pauli-Villars regularization, which introduces fictitious heavy particles whose contributions cancel those of genuine (virtual) particles at high momenta; and discretization of space-time on a lattice. At this point all predictions for physical quantities are finite, but cutoff dependent. One identifies a few of these cutoff dependent quantities – in QED, the charge (measured at some reference distance) $f_1(e_{\mathrm{bare}}, \mathrm{cutoff}, m_{\mathrm{bare}})$, and electron survival amplitude ("wave function renormalization") $f_2(e_{\mathrm{bare}}, \mathrm{cutoff}, m_{\mathrm{bare}})$, and also the electron mass $f_2(e_{\mathrm{bare}}, \mathrm{cutoff}, m_{\mathrm{bare}})$, and holds them equal to their physical values – namely, $e(r_{\mathrm{reference}})$, 1, $m_e$ – as the cutoff is removed. Then the big result, demonstrated in important example calculations by Schwinger, Feynman, and Tomonaga, and proved in general by Dyson, is that all other physical quantities, when expressed in terms of these three, approaches a finite limit as the cutoff is removed. Note that the $f_j$ do *not* have finite limits, but we've *renormalized* them to their correct physical values.

The details of regularization and renormalization are the heart and soul of the more advanced, most characteristic parts of quantum field theory. This is where one confronts the deeper implications of having an infinite number of degrees of freedom per unit volume, i.e. arbitrarily high momentum modes, in the theory. I'll just mention two major consequences, that we'll have occasion to refer to:

1. The *renormalization group*, in this context, is the statement that you can get the same theory after choosing different values of the reference distance (or momentum), as long as you compensate by using appropriate values of the physical quantities.

2. *Anomalies* can result when it is impossible for the cutoff to maintain a symmetry of the classical theory one is attempting to quantize. An anomalous symmetry, by definition, is a symmetry of the classical theory that is violated by the quantization procedure.

Feynman's approach to quantum electrodynamics was especially evocative and user-friendly. It introduced the apparatus of Feynman graphs, which have become a major part of the language of quantum physics.

## Weak Interaction

The need for an additional nuclear interaction, besides the dominant strong interaction, emerged from the study of radioactivity. Whereas $\alpha$ radioactivity can be understood as the emission of a helium nucleus by strong interactions (with electromagnetism playing an important role), and of course $\gamma$ radiation as electromagnetic de-excitation, involving the emission of hard photons, the processes of $\beta$ radioactivity exhibited several qualitatively different features, which indicated the need for an essentially new ingredient. Major distinctive aspects include:

1. $\beta$ radioactivity occurs with emission of an electron and (as interpreted by Pauli, to balance energy and momentum in the decays) an unobserved antineutrino. The neutrino has no electric charge, and is not produced in strong interactions – so its involvement requires a new interaction.

2. In $\beta$ radioactivity, the numbers of protons and of electrons are not separately conserved. In terms of atomic number $A$ and charge $Z$ the schema is

$$(A, Z) \rightarrow (A, Z+1) + e + \bar{\nu}_e \tag{1}$$

or in terms of underlying nucleons

$$n \rightarrow p + e + \bar{\nu}_e \tag{2}$$

In the language we developed earlier: the weak interaction violates strong isospin $I_3$.

3. $\beta$ decays are intrinsically much slower than other decays, indicating a much weaker strength of interaction. ($\alpha$ and $\gamma$ rays can be very slow, too, but in cases where that occurs it can be ascribed to tunneling barriers or to the existence of angular momentum barriers.)

When muons and charged pions were discovered, people quickly realized that their main decays

$$\mu \rightarrow e + \bar{\nu}_e + \nu_\mu \tag{3}$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{4}$$

shared major qualitative features with $\beta$ radioactivity. Thus a general "weak interaction" was identified.

Fermi made a major theoretical contribution, by applying the ideas of quantum field theory to the weak interaction. He formulated the decays as processes of particle creation and destruction, expanding upon the techniques used in quantum electrodynamics. Adopting the simplest possible form for the interactions (contact interaction – no field derivatives!), he got a good description of the energy spectrum in $\beta$ decay.

Detailed study of weak decays eventually converged upon the form

$$H_{\text{effective}} = \frac{G_F}{\sqrt{2}} J^\mu J^\dagger_\mu \tag{5}$$

where $G_F \approx 10^{-5}\text{GeV}^{-2}$ is the Fermi constant, the $\dagger$ indicates hermitean conjugate, and the factor $1/\sqrt{2}$ is an historical artifact. The weak current takes the form

$$J^\mu = J^\mu(\text{hadronic}) + J^\mu(\text{leptonic}) \tag{6}$$

where the leptonic current takes the form

$$\begin{aligned} J^\mu(\text{leptonic}) &\equiv V^\mu(\text{leptonic}) - A^\mu(\text{leptonic}) \\ &= \bar{e}\gamma^\mu(1-\gamma_5)\nu_e + \bar{\mu}\gamma^\mu(1-\gamma_5)\nu_\mu + \bar{\tau}\gamma^\mu(1-\gamma_5)\nu_\tau \end{aligned} \tag{7}$$

Here "V" and "A" stand for vector and axial vector respectively. The $V - A$ form of the weak current indicates that only left-handed helicity leptons (and right-handed helicity antileptons) participate in the weak current. It gives a precise version of the semi-empirical rule of "maximal parity violation". For the hadronic current, we now know that we should write a very similar form, but with an interesting wrinkle:

$$J^\mu(\text{hadronic}) = (\bar{d}, \bar{s}, \bar{b})\gamma^\mu(1-\gamma_5)U_{\text{CKM}}(u, c, t)^T \tag{8}$$

where $U_{\text{CKM}}$ is a 3 x 3 unitary matrix, essentially what is called the Cabibbo-Kobayashi-Maskawa matrix.

The saga of how these forms gradually emerged, at first from semi-phenomenological considerations and then increasingly from general theoretical principles, is fascinating, but nowadays a top-down approach – invoking the general principles, especially gauge symmetry – is an easier way to learn the subject, and ultimately more profound. Let me just remark that the leptonic part of the story got filled in very quickly following the discovery of parity violation, while the hadronic part at first involved all kinds of ingenious tricks and special hypotheses (*e. g.* conserved vector current, partially conserved axial current, $f$ and $d$ coefficients in the eightfold way (Cabibbo theory)) because people didn't have quarks – in the modern sense, as specific objects in quantum field theory – or QCD available.

The appearance of the CKM matrix seems, superficially, to be a distinct difference between the hadronic and leptonic charged currents. In reality, however, the difference is purely formal, and the different appearance is a matter of convenience and convention. In the hadronic case it is convenient to use the fields that create quarks of definite mass as our building-blocks, and then we must take into account that in the charged current the mass eigenstates for charge $\frac{2}{3}$ versus charge $-\frac{1}{3}$ quarks are misaligned. In the leptonic case it is convenient to use instead the fields that creates leptons of definite lepton number. Then the (approximate) laws of lepton number, which are observed to be well obeyed in most weak processes, are built in. For the charged leptons, the particles with definite lepton numbers also have definite mass. For neutrinos, however, the phenomenon of oscillations indicates precisely that the neutrinos of definite mass do not coincide with the neutrinos of definite lepton numbers. The relationship between those two neutrino concepts brings in a unitary transformation analogous to the CKM matrix. In the neutrino context, it is known as the PMNS matrix, or alternatively the MSW matrix. I'll spare you the politics behind these acronyms; let me just remark that the "M" and "S" are not really in common, despite appearances.

## Electroweak Theory

Higher-order corrections in the Fermi theory could not be fixed by the renormalization procedure that worked for quantum electrodynamics. One could start to do better – the divergences became less severe – by making the theory more like electrodynamics. Specifically, one could do better, and also "explain" the current x current form of the effective interaction, by postulating the existence of intermediate vector bosons. These would be photon-like particles, in the sense of being described by vector fields, but different in that the particles involved are massive and electrically charged. They are, of course, the modern $W^{\pm}$ bosons.

The fact that the $W$ bosons are charge already indicates, in the context of gauge theory, that the weak and electromagnetic interactions must be considered together. It is sometimes said that they must be (and have been) "unified", but that is misleading, since within the Standard Model there are still two independent gauge groups $SU(2) \times U(1)$, with two independent (and empirically unequal) couplings.

On the face of it gauge symmetry leads to vector bosons of zero mass, and therefore needs some modification to describe massive $W^{\pm}$ bosons. The Higgs mechanism, which we'll discuss in detail later, preserves the advantages of gauge theory – good high-energy behavior (renormalizability) and the decoupling of longitudinal modes[1]. The leading idea is to have the basic equations of the theory be gauge invariant, but to base the solution that governs the world on a non-invariant ground state, in which a gauge non-singlet but Poincare invariant scalar "condensate" fills space-time. With that mechanism in place, one can construct the modern $SU(2) \times U(1)$ electroweak theory in a few well-motivated strokes, as we shall. It contains, in addition to the $W^{\pm}$ bosons and of course the photon, a heavy neutral $Z$ boson, that mediates additional weak interactions.

Building on techniques originated by his thesis advisor Veltman, 'tHooft proved that non-abelian gauge theories are renormalizable, in a similar sense to electrodynamics. That made the theories more credible theoretically, and also supports many useful higher-order calculations. $K$ meson oscillations, at the quark level schematically

$$\bar{s}d \leftrightarrow \bar{d}s \tag{9}$$

are governed by second-order weak interactions. They provide an especially sensitive test case.

[1] Longitudinal modes are problematic, when we combine relativity and quantum mechanics. The quantum-mechanical commutators, if they're covariant, will be of opposite sign for space-like and time-like modes. For vector bosons we want the transverse (space-like) modes to be normal, so we need to get rid of the time-like mode, which we can do using gauge invariance.

## Applications and Frontiers

Today electroweak interaction theory is, for the most part, not an independent discipline, but a tool. There is a small community of specialists who do high-order calculations, mainly in QED, to do justice to the accuracy that experiments can attain. An outstanding example of this work is the calculation of magnetic dipole moments, both for the electron and for the muon. At present, we have the astonishingly accurate measurement

$$\frac{g-2}{2} \;=\; (11659209 \pm 6) \times 10^{-10} \tag{10}$$

There is an approximately $3\sigma$ discrepancy between theory and experiment, which might – or might not – indicate some the contribution of some new physics (*e. g.* supersymmetry). The calculations include all four-loop electromagnetic diagrams, and some classes of higher-order diagrams that are enhanced with powers of $\ln m_\mu/m_e$.

The weak interaction plays a vital role in astrophysics and cosmology, especially in connection with nuclear chemistry. The basic process that powers main sequence stars is a chain of reactions that cook protons into $He^4$. Balancing charge and electron number gives us the basic scheme

$$4p \;\rightarrow\; (ppnn) + 2e^+ + 2\nu_e \tag{11}$$

Since $He^4$ is a highly favorable nucleus, energetically – it has a large "mass defect" or binding energy – this is exothermic. Evidently, two weak interaction steps are involved. One also has nuclear cooking in during the first few minutes of the big bang, leading to a successful account of the relative abundances of $H^1, H^2, He^3, He^4, Li^7$.

There is a very active effort to pin down the PMNS/MSW matrix in neutrino physics, as well as the neutrino masses. Unfortunately, theory provides little useful guidance in this endeavor, and we don't really know what we will do with the information, once we've got it.

Similarly to what we described for QCD, with an excellent quantitative theory of electroweak interactions available, we can search for small discrepancies. Important frontiers include the search for fundamental *electric* dipole moments, as we mentioned earlier, together with lepton-number violating decays such as $\mu \rightarrow e\gamma$, and various heavy quark decay processes.

Possibly the most profound "application" of the weak interaction is that it gives important guidance in forming unified field theories. The idea of spontaneous symmetry breaking, the importance of underlying massless fermions (chirality), and of course the centrality of gauge symmetry, are all lessons we take from the electroweak theory. The complex pattern of weak isospin and hypercharge assignments is data that we can aspire to organize and explain through unification ideas. And in fact, as we'll discuss, we can not merely aspire to do those things, but actually do them.